Estimating Extrudate Rheological Properties

There are not many ways to gather rheological data for extrusion systems flash steam at the die. The temperatures remove most analytic equipment from choices for measurements. One approach that is used and widely recognized in the literature is use of a slit-die rheometer or a capillary die rheometer. This is a good choice, if you have access to this type of die.

In most cases, this type of die is not readily available. It is possible to use an existing die to get a good estimation of extrudate's rheological properties. This is done by taking the equations for a rheological model and creating a form where it can be solved for the variables of interest.

One equation that can be used as a starting point is equation 3.11 in Extrusion of Foods, Volume 1, by Judson Harper, page 26. That equation is:

$$\eta=\eta^{*}\dot{\gamma}^{n-1}\exp\left(\frac{\Delta E_{\eta}}{RT}\right)\exp(KM)$$

where:
η = apparent viscosity
η^{*} = reference apparent viscosity
ΔE_{η} = activation energy for flow
R = gas constant
T = absolute temperature of the extrudate
K = moisture constant
M = moisture content, fraction (dry basis moisture)

The next step is to simplify the equation for further manipulation. The substitutions to be used are:

$$A=\frac{\Delta E_{\eta}}{R} \quad \text{and}$$

$$B=K \quad \text{which gives:}$$

$$\eta=\eta^{*}\dot{\gamma}^{n-1}e^{\left(\frac{A}{T}\right)}e^{BM}$$

The equation above gives a relationship for apparent viscosity as a function of the reference viscosity, shear rate, temperature, and moisture content.

Using the knowledge that "…viscosity is directly proportional to the pressure drop across the capillary divided by the flow rate through the system." (page 59 in "Rheological Methods in Food Process Engineering", 2nd Edition by James F. Steffe, link), we can define:

$$\eta C_{1}=\frac{\Delta P}{Q}$$

Q = volumetric flow rate
C_1 = proportionality constant.

The equation can be rearranged to: $\Delta P = C_1 Q \eta$

Substituting η from the modified Harper's equation, we get:

$$\Delta P = C_1 Q \left[\eta^* \gamma^{n-1} e^{\left(\frac{A}{T}\right)} e^{BM} \right]$$

Since shear rate is directly proportional to volumetric flow rate, we know that

$$\dot{\gamma}^{n-1} = C_2 Q^{n-1}$$

C_2 = the proportionality constant relating the shear rate and volumetric flow rate.

We now have:

$$\Delta P = C_1 Q \left[\eta^* C_2 Q^{n-1} e^{\left(\frac{A}{T}\right)} e^{BM} \right]$$

This can be expressed as:

$$\Delta P = C_1 C_2 \eta^* Q^n \left[e^{\left(\frac{A}{T}\right)} e^{BM} \right]$$ note that η^* (the reference apparent viscosity) is a constant.

Taking the natural log of the equation results in:

$$\ln(\Delta P) = \ln\left(C_1 C_2 \eta^* Q^n \left[e^{\left(\frac{A}{T}\right)} e^{BM} \right] \right) = \ln\left(C_1 C_2 \eta^*\right) + n \ln(Q) + \frac{A}{T} + BM$$

Note that $\ln(C_1C_2\eta^*)$ is a constant. It will shift a line up or down on a log-log chart, but not change the slope of the line.

If an experiment is run where the flow rate, temperature, and moisture content of the extrudate is changed, data can be generated for regression using the above equation. That regression will give n, A, and B. From that data and the geometry of the die assembly, the value of η^* can be calculated either manually or by use of a computational fluid dynamics program.

The temperature to be measured is the temperature of the extrudate. A tip sensitive temperature probe submerged in the flowing extrudate will give a reasonable measure of the extrudate temperature. **Barrel temperature is generally not a good measure of the extrudate temperature.**

As with any experimental data, using the results to predict flow behavior outside the ranges in the regression data is questionable.